**Al-Farabi KAZAKH NATIONAL UNIVERSITY**

**Faculty of mechanics and mathematics**

**Educational program for the specialty «5В060100-Mathematics»**

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|  | Approved by the Faculty Scientific Council meetingProtocol №\_\_\_ from \_\_\_\_\_\_\_\_\_\_\_\_ 2014 **Dean of the faculty**  **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_A. Kadyrbekuly** |

**SULLABUS**

**Module No. \_\_\_ “\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_”**

**«\_\_\_\_\_\_\_\_\_» «Calculus of variations and optimization methods»**

3 course, k/s, r/s, autumn semester, 3 credits

**Lecturer**:

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**Aim and problems of the course:**

**Aim:**

The analysis of the classical variational problems and optimization control problems.

**Problems:**

The consideration of the methods of the classical variational problems and optimization control problems. The application of these methods for solving practical variational problems and optimization control problems.

**Competences:**

Knowledge of methods of solving for variational problems and optimization control problems.

**Pre Essential Elements**:

Differential equations. Mathematical analysis. Mechanics.

**Post** **Essential Elements**:

Numerical methods for extremum problems. Inverse problems. Optimization control problems for distributed systems.

**STRUCTURE AND CONTENT OF THE COURSE**

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| --- | --- | --- | --- |
| **week** | **subject** | **hours** | **maximal mark** |
| **Module 1. Introduction** | | | |
| **1** | **Lecture 1. Practical examples of the extremum problems**. Maximization of the flight of the body. Brachistochrone problem. Maximization of the flight of the missile. | 2 | 0 |
| **Practical work 1**. Practical examples of the extremum problems. | 1 | 3 |
| **Homework 1**. Practical examples of the extremum problems. | 1 | 10 |
| **2** | **Lecture 2. Minimization of functions**. Stationary condition. Examples. Maximization of the flight of the body. Minimization of the function of many variables. | 2 | 1 |
| **Practical work 2**. Minimization of functions and stationary condition. | 1 | 3 |
| **Homework 2**. Use stationary condition for the concrete function*.* Check the properties of the stationary points. Chose the function with given property. | 1 | 10 |
| **Module 2. Calculus of variations** | | | |
| **3** | **Lecture 3. Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light low. | 2 | 2 |
| **Practical work 3**. Euler equation for Lagrange problem. | 1 | 3 |
| **Homework 3.** Determine Euler equation for the concrete Lagrange problem. Find the general solution of Euler equation, which depends from two constants. Find these constants by means of the given boundary conditions. Find the corresponding solution of the boundary problem. Calculate the corresponding value of the given functional. Calculate the value of the given functional for the linear function which satisfies the given boundary conditions. Compare these results. | 1 | 10 |
| **4** | **Lecture 4. Lagrange problem for the functions family.** Problem statement. The system of Euler equations. Example. Principle of the least action. | 2 | 1 |
| **Practical work 4**. Lagrange problem for the functions family. | 1 | 3 |
| **Homework 4**. Determine the system of Euler equations for the concrete problem. Find general solution of this system. Find the solution of Euler equations, which satisfies boundary conditions. Show the graphs of these solutions. Calculate the corresponding value of the given integral. | 1 | 10 |
| **5** | **Lecture 5. Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam. | 2 | 2 |
| **Practical work 5**. Lagrange problem with high derivatives. | 1 | 3 |
| **Homework 5**. Determine the system of Euler – Poisson equation for the concrete problem. Find general solution of this equation. Find the solution of Euler – Poisson equation, which satisfies given boundary conditions. Show the graph of this solution. Calculate the corresponding value of the given integral. | 1 | 10 |
| **6** | **Lecture 6. Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string. | 2 | 1 |
| **Practical work 6.** Lagrange Problem for functions with many variables. | 1 | 3 |
| **Homework 6.** Determine Ostrogradsky equation for the concrete problem. | 1 | 10 |
| **7** | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | 2 | 2 |
| **Practical work 7.** Bolza Problem. | 1 | 3 |
| **Homework 7.** Determine Euler equation and the transversality conditions for the concrete problem. Find the general solution of this equation. Find the solution of boundary problem. Calculate the corresponding value of the given integral. | 1 | 10 |
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| **Border control 1** |  | **100** |
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| **8** | **Lecture 8. Variational problems with isoperimetric conditions.** Problems with isoperimetric condition. Lagrange multipliers method. A spectrum problem. The problem with many isoperimetric conditions. | 2 | 2 |
| **Practical work 8.** Variational problems with isoperimetric conditions. | 1 | 3 |
| **Homework 8.** Determine Euler equation for the concrete problem. Verify the sign of the Lagrange multiplier with using multiplication of Euler equation by unknown function and integration. Find the general solution of Euler equation; it depends from two constants and Lagrange multiplier. Using given boundary conditions and isoperimetric condition find three unknown constants. Find the set of the solutions of the conditions of the extremum. Calculate the value of the given integral for all solution of the conditions of the extremum. | 1 | 8 |
| **9** | **Lecture 9. Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum. | 2 | 1 |
| **Practical work 9.** Variational problems with pointwise constraints. | 1 | 3 |
| **Homework 9.** Denote the system of the extremum conditions (concrete Euler equations with boundary and addition conditions). Multiply the first Euler equation by the given parameter a, and second equation by b. Add these equalities with using of the condition (\*). Find Lagrange multiplier λ. Put λ to Euler equations. Find the general solutions of two Euler equations. It equals the sum of the general solution of the corresponding homogeneous equation and the constant, which satisfies the given equation. Find four constants from general solutions of Euler equations with using of the boundary conditions. Put these constants to the formulas of the general solutions. It will be the solution of the problem. | 1 | 8 |
| **Module 3. Optimization control problems** | | | |
| **10** | **Lecture 10. Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions. | 2 | 1 |
| **Practical work 10.** Easiest optimization control problems. | 1 | 3 |
| **Homework 10.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. | 1 | 8 |
| **11** | **Lecture 11. Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving). | 2 | 2 |
| **Practical work 11.** Optimization control problems for the vector case. | 1 | 3 |
| **Homework 11.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. | 1 | 8 |
| **12** | **Lecture 12. Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method. | 2 | 1 |
| **Practical work 12.** Optimization control problem with fixed final state. | 1 | 3 |
| **Homework 12.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality with using of firing method. | 1 | 8 |
| **13** | **Lecture 13. Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals. | 2 | 2 |
| **Practical work 13.** Differentiation of functionals and abstract optimization problems. | 1 | 3 |
| **Homework 13.** Calculate Gateau derivative for the concrete functional. Determine gradient method and projection gradient method. | 1 | 8 |
| **14** | **Lecture 14**. **Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional. | 2 | 1 |
| **Practical work 14.** Variational inequalities. | 1 | 3 |
| **Homework 14.** Calculate Gateau derivative for the concrete functional. Determine variational inequality. Find the solution of variational inequality. Calculate the value of minimizing functional. | 1 | 8 |
| **15** | **Lecture 15. Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example. | 2 | 2 |
| **Practical work 15.** Existence and uniqueness of extremum problems. | 1 | 3 |
| **Homework 15.** Prove the convexity and continuity of the concrete functional. Prove the convexity, closeness and boundedness of the concrete set of admissible control. Prove the existence of the optimal control. | 1 | 8 |
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| **Border control 2** |  | **100** |
|  | **Examination** |  | **100** |
|  | **TOTAL** |  | **100** |

**LITERATURE**

**Basic:**

1. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 1979.
2. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. – [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) .
3. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011.
4. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001.
5. Эльсгольц Л.Э. Дифференциальные уравнения и вариационное исчисление. – М., Наука, 1969.
6. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. <http://www.amazon.com/Optimal-Control-Theory-Introduction-Engineering/dp/0486434842>
7. Serovajsky S. Practical Course of the Optimal Control Theory with Examples. Almaty, Қазақ университеті, 2011.

**Additional:**

1. Габасов Р., Кириллова Ф. Качественная теория оптимальных процессов. – М., Наука, 1907. – 507 с.
2. Иоффе А. Д., Тихомиров В. М. Теория экстремальных задач. – М., Наука, 1974. – 480 с.
3. Канторович Л. В., Акилов Г. П. Функциональный анализ. – М., Наука, 1977. – 744 с.
4. Ахиезер Н.И. Лекции по вариационному исчислению. – М., ГИТТЛ, 1956.
5. Гельфанд И.М., Фомин С.В. Вариационное исчисление. – М., Физматгиз, 1961.
6. Краснов М.Л., Макаренко Г.И, Киселев А.И. Вариационное исчисление. – М., Наука, 1973.
7. Лаврентьев М.А., Люстерник Л.А. Основы вариационного исчисления. Том 2. – М., ОНТИ, 1935.
8. Miersemann E. Calculus of Variations. Lecture Notes. – Leipzig, 2012.

АКАДЕМИЧЕСКАЯ Политика курса

Все виды работ необходимо выполнять и защищать в указанные сроки. Студенты, не сдавшие очередное задание или получившие за его выполнение менее 50% баллов, имеют возможность отработать указанное задание по дополнительному графику. Студенты, пропустившие лабораторные занятия по уважительной причине, отрабатывают их в дополнительное время в присутствии лаборанта, после допуска преподавателя. Студенты, не выполнившие все виды работ, к экзамену не допускаются. Кроме того, при оценке учитывается активность и посещаемость студентов во время занятий.

будьте толерантны, уважайте чужое мнение. Возражения формулируйте в корректной форме. Плагиат и другие формы нечестной работы недопустимы. Недопустимы подсказывание и списывание во время сдачи СРС, промежуточного контроля и финального экзамена, копирование решенных задач другими лицами, сдача экзамена за другого студента. Студент, уличенный в фальсификации любой информации курса, несанкционированном доступе в Интранет, пользовании шпаргалками, получит итоговую оценку «F».

За консультациями по выполнению самостоятельных работ (СРС), их сдачей и защитой, а также за дополнительной информацией по пройденному материалу и всеми другими возникающими вопросами по читаемому курсу обращайтесь к преподавателю в период его офис-часов.

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| --- | --- | --- | --- |
| Letter mark | Number mark | % | Traditional mark |
| А | 4,0 | 95-100 | Very good |
| А- | 3,67 | 90-94 |
| В+ | 3,33 | 85-89 | Good |
| В | 3,0 | 80-84 |
| В- | 2,67 | 75-79 |
| С+ | 2,33 | 70-74 | Satisfactory |
| С | 2,0 | 65-69 |
| С- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D- | 1,0 | 50-54 |
| F | 0 | 0-49 | Non satisfactory |
| I  (Incomplete) | - | - | «The course is not finished»  (*do not take into consideration of GPA)* |
| P  (Pass) | **-** | **-** | «given a credit»  (*do not take into consideration of GPA)* |
| NP  (No Рass) | **-** | **-** | «did not give a credit»  (*do not take into consideration of GPA)* |
| W  (Withdrawal) | - | - | «renunciation of the course»  (*do not take into consideration of GPA)* |
| AW  (Academic Withdrawal) |  |  | Renunciation of the course by academic cause  (*do not take into consideration of GPA)* |
| AU  (Audit) | - | - | «the course is listen»  (*do not take into consideration of GPA)* |
| Атт. |  | 30-60  50-100 | Attested |
| Не атт. |  | 0-29  0-49 | No attested |
| R (Retake) | - | - | Retake the course |

Session № \_\_ of « \_\_ » \_\_\_\_\_\_\_\_\_\_\_ 2013.

**Head of the Department S. Muhambetzhanov**

**Lecturer S. Serovajsky**